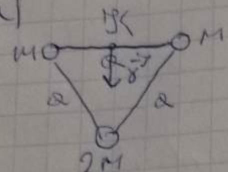
12.1

Oblicz wartość natężenia pola grawitacyjnego w punkcie "K" w sytuacjach pokazanych na rysunkach. Na każdym rysunku zaznacz zwrot natężenia pola w punkcie "K". Do obliczeń przyjmij $m = 1{,}2\,kg$, $a = 1\,m$

a)



$\gamma = \frac{GM}{r^2}$

$r = \frac{\sqrt{3}}{2} a$

$M = 2m$

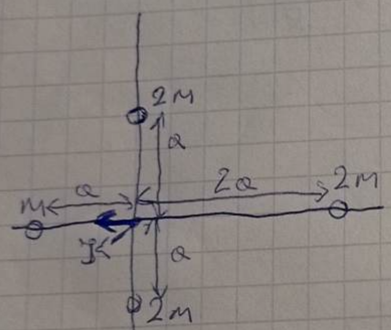
$\gamma = \frac{G \cdot 2m}{(\frac{\sqrt{3}}{2} a)}$

$\gamma = \frac{8}{3} \frac{GM}{a^2}$

podstawiamy

$\gamma = \frac{8}{3} \cdot \frac{6{,}67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \cdot 1{,}2\,kg}{(1\,m)^2} \approx 2 \cdot 10^{-10} \frac{N}{kg}$

b)



wiemy że:

$M_1 = 2m \quad r_1 = a$

$M_2 = 2m \quad r_2 = 2a$

$M_3 = 2m \quad r_3 = a$

$M_4 = m \quad r_4 = a$

czyli

$\gamma = \frac{GM}{a^2}$

$\gamma_1 = \frac{GM_1}{r_1^2} = \frac{G2m}{a^2} = 2 \cdot \frac{GM}{a^2} = 2\gamma$

$\gamma_2 = \frac{GM_2}{r_2^2} = \frac{2Gm}{4a^2} = \frac{1}{2} \cdot \frac{Gm}{a^2} = \frac{1}{2}\gamma$

$\gamma_3 = \frac{GM_3}{r_3^2} = 2\frac{Gm}{a^2} = 2\gamma$

$\gamma_4 = \frac{GM_4}{r_4^2} = \frac{G \cdot m}{a^2} = \gamma$

$\gamma_{pion} = \gamma_1 - \gamma_3 = 0$

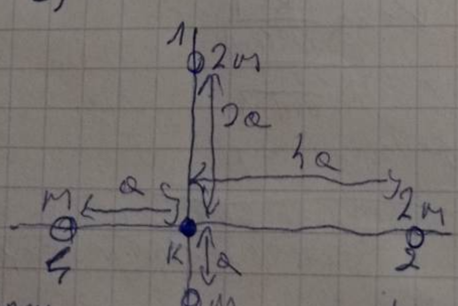
$\gamma_{poz} = \gamma_4 - \gamma_2 = \frac{1}{2}\gamma$

podstawiamy

$\gamma = \frac{1}{2} \cdot \frac{6{,}67 \cdot 10^{-11} \cdot 1{,}2\,kg}{(1\,m)^2} \approx 4 \cdot 10^{-11} \frac{N}{kg}$

$\gamma = \gamma_{poz} = \frac{1}{2} \frac{Gm}{a^2}$

c)



$M_1 = 2m \quad r_1 = 2a$

$M_2 = 2m \quad r_2 = 4a$

$M_3 = m \quad r_3 = a$

$M_4 = m \quad r_4 = a$

$\gamma_1 = \frac{GM_1}{r_1^2} = \frac{2GM}{4a^2} = \frac{1}{2}\frac{GM}{a^2} = \frac{1}{2}\gamma$

$\gamma_2 = \frac{GM_2}{r_2^2} = \frac{2GM}{16a^2} = \frac{1}{8}\frac{GM}{a^2} = \frac{1}{8}\gamma$

$\gamma_3 = \frac{GM_3}{r_3^2} = \frac{GM}{a^2} = \gamma$

$\gamma_4 = \frac{GM_4}{r_4^2} = \frac{GM}{a^2} = \gamma$

z pitagorasa

$\gamma_w^2 = \gamma_{13} + \gamma_{24}$

wiemy też

$\gamma_{13} = \gamma_1 - \gamma_3$

$\gamma_{24} = \gamma_2 - \gamma_4$

czyli:

$\gamma_w^2 = (\gamma_1 - \gamma_3)^2 + (\gamma_2 - \gamma_4)^2$

$\gamma_w = \sqrt{(\gamma_1 - \gamma_3)^2 + (\gamma_2 - \gamma_4)^2}$

$\gamma_w = \frac{\sqrt{65}}{8} \cdot \frac{GM}{a^2}$

podstawiamy

podstawiamy

$= \frac{\sqrt{65}}{8} \cdot \frac{6{,}67 \cdot 10^{-11} \cdot 1{,}2\,kg}{(1\,m)^2}$

$\approx 8 \cdot 10^{-11} \frac{N}{kg}$